

EECS 562
Homework 9

1. (Concepts: Instantaneous phase and frequency) Determine the instantaneous phase and frequency for the following signals. Let $f_c=200\text{kHz}$
 - a. $\cos(2\pi f_c t + 400t^2)$
 - b. $\cos(2\pi f_c t + 400\sqrt{t})$
2. (Concept: FSK in time domain) Given a set of information bits $b_i = \{1, 0, 0, 1\}$.
Let $x(t) = 0$ for 2 ms for a bit = 0 and $x(t) = 2$ for 2ms for a bit = 1, i.e., the bit time is 2 ms.
A modulated RF signal is $y_{\text{RF}}(t) = 10\cos(2\pi(x(t)*10000 + f_c)t)$ where $f_c=10\text{kHz}$. Plot $y(t)$ for $0 < t < 4\text{ms}$.
3. (Concepts: Recognizing the form of different analog modulations, bandwidth and power in FM signals) A message signal is $x(t) = \cos(2\pi f_m t)$. The transmitted RF signal is given by $y_{\text{RF}}(t) = 10\cos(2\pi f_c t + \beta \sin(2\pi f_m t))$, where $f_c=100\text{MHz}$, $\beta=2$, and $f_m=15\text{kHz}$.
 - a. Is the RF modulation:
 - i. PM
 - ii. FM
 - iii. VSB
 - iv. SSB
 - v. DSB-LC
 - b. What is the instantaneous frequency?
 - c. What is the total power in dB_W ?
 - d. What is the frequency deviation, Δf ?
 - e. What is the RF bandwidth of $y_{\text{RF}}(t)$?
 - f. How much power is at 100 MHz in dB_W ?
 - g. How much power is at 100.03 MHz in dB_W ?
4. (Concepts: Recognizing the form of different analog modulations, spectrum, bandwidth and power in FM signals) Let the RF signal be
 $y_{\text{RF}}(t) = A_c \cos(\theta_i(t))$ where
 $\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$ with $x_{\text{bb}}(t) = A_m \cos(2\pi f_m t)$
Here $A_m=2.0\text{V}$ and $f_c=109\text{MHz}$, $f_m=1\text{kHz}$, $A_c=20\text{V}$, $\beta=0.4$
 - a. Is the phase or frequency modulation?
 - b. What is the modulation index?
 - c. Find the frequency deviation.
 - d. Find the frequency sensitivity factor in Hz/V .
 - e. Plot the amplitude spectrum of $y_{\text{RF}}(t)$.
State any approximations.
 - f. What is the total power in $y_{\text{RF}}(t)$ in dB_W ?
 - g. What is the RF bandwidth?
5. (Concepts: Parameters of FM signals and the spectrum, bandwidth and power in FM signals) Let the RF signal be
 $y_{\text{RF}}(t) = A_c \cos(\theta_i(t))$ where
 $\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$ with $x_{\text{bb}}(t) = A_m \cos(2\pi f_m t)$
Here $A_m=1.0\text{V}$ and $f_c=109\text{MHz}$, $f_m=1\text{kHz}$, $A_c=10\text{V}$, $\beta=4$.

- a. Find the frequency deviation, Δf .
 - b. Find the frequency sensitivity factor in Hz/V.
 - c. Plot the amplitude spectrum of $y_{RF}(t)$.
 - d. What is the RF bandwidth?
6. (Concept: Carson's rule to find the bandwidth of FM signals) For each case below use using Carson's rule to find the bandwidth of the frequency modulated signal. The deviation constant = 0.75 Hz/volt.
- a. $A_c=1.5V$ and $f_m=2$ Hz message amplitude = $A_m=0.5V$,
 - b. $A_c=1.5V$ and $f_m=2$ Hz message amplitude = $A_m=1.5V$
 - c. $A_c=1.5V$ and $f_m=0.5$ Hz, message amplitude = $A_m=1.5V$
 - d. Define a bandwidth expansion factor as B_{RF} / B_{bb} . Calculate the bandwidth expansion factor the systems defined in parts a)-c).
 - e. Use Interactive: Spectrum of FM/PM Tone Modulated Signals to verify your answers to a-c and report magnitude spectrum plots for each case.
7. (Concept: M-ary FSK) Let the message signal $m(t)$ given below be the input to an FM modulator with $k_f=15\text{Hz/V}$ and $f_c=200$ Hz and $A_c=1.0$. $v = \{2.5, 5, 7.5\}$, the symbol time=40ms.
- $$m(t) = \sum_{i=1}^3 v_i \text{rect}\left(\frac{t-(i-1)T_s-0.5T_s}{T_s}\right)$$
- a. Plot $m(t)$.
 - b. Plot the RF signal, $y_{RF}(t)$.
 - c. What frequencies are present in $y_{RF}(t)$
 - d. Suggest a detector architecture detect each symbol
8. (Concept: Orthogonal M-ary FSK) In a stream of bits each pair of bits (2 bits) is mapped into one voltage level to form the baseband signal, e.g. $m(t) = v_i$ for 20 ms, where $v_i=0, 2.5, 7.5, 10$. The message signal is input to an FM modulator with $k_f=40\text{Hz/V}$ and $f_c=2000$ Hz and $A_c=1$.
- a. What is the bit rate for this signal?
 - b. During a symbol time of 20ms are the 4 possible transmitted RF signals, are these orthogonal to each other?
9. (Concept: Characteristics of FM balanced discriminator) A DC blocking capacitor is not needed when a balanced discriminator is used to demodulate FM signals, why?

10. (Concept: The non-linearity of FM modulation) A message signal $m(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$ is input to an FM modulator with modulation index β . The transmitted FM signal can be written as

$$s(t) = A_c \cos\left(2\pi f_c t + \beta \left(\frac{A_1}{2\pi f_1} \sin(2\pi f_1 t) + \frac{A_2}{2\pi f_2} \sin(2\pi f_2 t)\right)\right) \text{ let}$$

$$\beta_1 = \beta \frac{A_1}{2\pi f_1} \quad \text{and} \quad \beta_2 = \beta \frac{A_2}{2\pi f_2} \quad \text{then } s(t) \text{ can be expanded using Bessel functions to form}$$

$$s(t) = A_c \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_k(\beta_1) J_n(\beta_2) \cos(2\pi (f_c + k f_1 + n f_2) t)$$

Given this knowledge is FM a linear or non-linear modulation and justify your answer.